EVALUATION OF THE ACCURACY OF A LOWER CONFIDENCE LIMIT ESTIMATE FOR SERIES SYSTEM RELIABILITY

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ABSTRACT

The purpose of this study is to evaluate the accuracy of a procedure used to compute an estimate of the lower 100(1-γ)% confidence limit for reliability of a system of independent components connected in logical series. The procedure takes a Bayesian approach and uses test data on the individual components where the sample sizes may be unequal and no knowledge of the component failure distribution is needed. A computer simulation is used to generate test failure data and to compute estimates for the lower 100(1-γ)% confidence limit on system reliability.
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I. INTRODUCTION

The problem of determining economically a lower \(100(1-\gamma)\%\) confidence limit on the system reliability of complex and expensive systems has plagued reliability managers for a long time. In 1968 Joseph Bram of the Center for Naval Analyses developed an approximate procedure to attack this problem for systems of independent components connected in logical series.\(^1\) It is the purpose of this study to test the accuracy of Bram's procedure and to determine the limits of its usefulness, especially in applications involving systems of ten or more components. The accuracy measurements are determined through a computer simulation.

In an effort to demonstrate reliability goals on expensive systems, it is usually not economically feasible to test the entire system many times. Bram takes a Bayesian approach using test data, successes and failures, on individual components or subsystems to obtain an estimate for the lower confidence limit on the overall system reliability. The method assumes that the components are independent, and test sample sizes for the various components may be unequal. No knowledge or assumptions about the components' failure rates is needed for the computation, only the test failure data.

\(^1\)Center for Naval Analyses, O.E.G. Research Contribution No. 79, Confidence Limits for System Reliability, by Joseph Bram, 2 February 1968.
II. RELIABILITY MODEL AND COMPUTATION PROCEDURES

A. THE RELIABILITY MODEL AND COMPUTATION PROCEDURE: METHOD A

1. Summary of the Reliability Model

Consider \( k \) independent components, as in Figure 1, connect in logical series with respective individual reliabilities \( p_1, p_2, \ldots, p_k \). By the product rule for series systems, the actual overall system reliability, \( R_s \), is given by

\[
R_s = \prod_{i=1}^{k} p_i
\]

![Series System Model](image)

Component reliability: \( p_1 \quad p_2 \quad \ldots \quad p_k \)
Number of tests: \( n_1 \quad n_2 \quad \ldots \quad n_k \)
Number of successes: \( s_1 \quad s_2 \quad \ldots \quad s_k \)

FIGURE 1.
THE SERIES SYSTEM MODEL

In Bram's Bayesian approach to finding an estimate for the lower confidence bound on system reliability, the \( p_i \)'s, and hence \( R_s \), are considered to be random variables. If the distribution from which the \( p_i \)'s were sampled were known, a lower confidence bound could be calculated without
testing. Since this distribution is not known however, the Bayesian's assume a prior distribution which when combined with the test data leads to a posterior distribution, hopefully one that is recognizable and from which confidence limits can easily be computed.

By considering the random variable

$$u = -\ln R_s = - \sum_{i=1}^{k} \ln p_i$$

the procedure approximates the density of $u$ by

$$\phi(u) = \frac{\beta^{a+1}}{a!} u^a e^{-\beta u}$$

The parameters $\alpha$ and $\beta$ will be estimated from test data. This density results from having chosen a prior density for $p_i$ which is questionable on the grounds that it is not normalizable (see parameterization of the prior density, Ref. 1) and leads to a posterior which is also not normalizable. The choice of priors causes difficulty in estimating the parameters of $\phi(u)$ when no failures occur on any of the components, and its effects will be investigated later in this study. However, since this is an approximate method, the development can proceed from the assumption that $u$ does in fact possess the above density and that estimates for $\alpha$ and $\beta$ can be determined.

If a new random variable $v$ is defined where

$$v = 2\beta u$$
then \( v \) is distributed "chi square" with \( 2(a+1) \) degrees of freedom. Then the lower \( 100(1-\gamma)\% \) confidence limit, \( R_{s,L(\gamma)} \), can be found as follows:

\[
P[2\beta u < \chi^2_{2(a+1),1-\gamma}] = 1 - \gamma
\]

\[
P[u < \frac{\chi^2_{2(a+1),1-\gamma}}{2\beta}] = 1 - \gamma
\]

\[
P[e^{-u} > \exp(-\frac{\chi^2_{2(a+1),1-\gamma}}{2\beta})] = 1 - \gamma
\]

\[
P[R_s > \exp(-\frac{\chi^2_{2(a+1),1-\gamma}}{2\beta})] = 1 - \gamma
\]

From the probability statement above,

\[
R_{s,L(\gamma)} = \exp(-\frac{\chi^2_{2(a+1),1-\gamma}}{2\beta})
\]

is the desired approximate lower confidence limit. The problem is to find an estimate, \( \hat{R}_{s,L(\gamma)} \), for \( R_{s,L(\gamma)} \) which will yield values sufficiently accurate for the reliability manager's purposes. Bram's method specifies estimates \( \hat{a} \) and \( \hat{\beta} \), which are computed from the test data, and substituted into the above expression, to obtain

\[
\hat{R}_{s,L(\gamma)} = \exp(-\frac{\chi^2_{2(\hat{a}+1),1-\gamma}}{2\hat{\beta}})
\]

In the following section Bram's computation procedure for \( \hat{a} \) and \( \hat{\beta} \) is described.
2. Description of the Computation Procedure

In the following, Bram's procedure for computing \( \hat{R}_{s,L}(\gamma) \), will be referred to as method A, and the author's modifications to the original procedure will be referred to as methods B and C.

For descriptive purposes, after testing has been done, consider the \( k \) system components to be arranged as follows: 1, 2, ..., \( k_1 \), \( k_1+1 \), ..., \( k \), where components 1, ..., \( k_1 \) are those which experience no failures, and \( k_1+1 \), ..., \( k \) are those which experience one or more failures. Let \( n_i \) be the number of tests on component \( i \) and \( s_i \) be the number of successes in the \( n_i \) tests. To get \( \hat{R}_{s,L}(\gamma) \) by method A proceed as follows:

\[
\hat{M}_A = \sum_{i=k_1+1}^{k} \sum_{j=s_i}^{n_i-1} \frac{1}{j},
\]

\[
\hat{V}_A = \sum_{i=k_1+1}^{k} \sum_{j=s_i}^{n_i-1} \frac{1}{j^2},
\]

\[
\hat{\alpha} = \frac{\hat{M}_A^2}{\hat{V}_A} - 1 \quad \text{and} \quad \hat{\beta} = \frac{\hat{M}_A}{\hat{V}_A}, \quad \text{and}
\]

\[
\hat{R}_{s,L}(\gamma) = \exp\left(-\frac{\chi^2(\hat{\alpha}+1),1-\gamma)}{2\hat{\beta}}\right)
\]

\( \hat{M}_A \) and \( \hat{V}_A \) are estimates of the mean and variance of \( u \) under method A. In cases where \( (\hat{\alpha}+1) \) is non-integer, interpolation is to be used.
3. **Shortcomings of Method A**

It should be noted that the double sums do not account for components which experience no failures in testing, and if none of the system's components fails, $\hat{\alpha}$ and $\hat{\beta}$ are not defined: under these circumstances the procedure can not be used as it stands. Herein lies the major obstacle to application of the procedure to highly reliable system testing programs where it is quite possible that no failures will occur. In testing the accuracy of method A, whenever no failures occur, $\hat{R}_{s,L(\gamma)}$ is set equal to unity by the author's choice, since Bram makes no allowance for this possibility.

Secondly, as $\hat{\alpha}$ is defined, it is possible that $2(\hat{\alpha}+1)$, the estimate of the degrees of freedom of the chi square random variable can be less than one. Since a chi square variate must have at least one degree of freedom, some provision must be made to account for this inconsistency. For computation purposes in testing the original procedure of method A and the modifications, whenever $2(\hat{\alpha}+1)$ is less than one, the degrees of freedom are set equal to one by the author's choice. For all fractional values of $2(\hat{\alpha}+1)$ greater than one, linear interpolation between the tabulated integer degrees of freedom are used, as specified in the original procedure.

B. **FIRST MODIFICATION TO THE COMPUTATION PROCEDURE:**

**METHOD B**

The only way that the true lower confidence limit $R_{s,L(\gamma)}$ can be unity is to have no failures on an infinite number of
tests; this would be impossible to demonstrate. In searching for a more realistic value for $\hat{R}_{s,L(Y)}$ when no failures occur, one might be led to believe that $\hat{R}_{s,L(Y)}$ should be close to but not equal to one. Due to its discrete character, the procedure of method A will compute one value of $\hat{R}_{s,L(Y)}$ which is closest to unity; this will happen when one failure occurs on the component with the greatest number of tests. An approximation of a more accurate estimate of $\hat{R}_{s,L(Y)}$ when no failures occur, can be obtained by using one half the original estimates $\hat{M}_A$ and $\hat{V}_A$, where one failure occurs on the component with the most tests. This has the effect of considering a partial failure on the component with the most tests and tends to smooth the discrete distribution of values of $\hat{R}_{s,L(Y)}$ near unity.

In summary, computations under method B proceed as follows:

$$\hat{M}_B = \hat{M}_A$$
$$\hat{V}_B = \hat{V}_A$$

if $k_1 < k$, and

$$\begin{align*}
\hat{M}_B &= \frac{1}{2(N_{\max} - 1)} \\
\hat{V}_B &= \frac{1}{2(N_{\max} - 1)^2}
\end{align*}$$

if $k_1 = k$.

$N_{\max}$ is the number of tests on the component with the greatest number of tests.
\[ \hat{\alpha} = \frac{\hat{M}^2}{\hat{V}} - 1 , \]

\[ \hat{\beta} = \frac{\hat{M}}{\hat{V}} , \] and

\[ \hat{R}_{s,L(\gamma)} = \exp\left[-\frac{\chi^2_{2(\hat{\alpha}+1),1-\gamma}}{2\hat{\beta}}\right] \]

If \( 2(\hat{\alpha}+1) < 1.0 \), then set \( 2(\hat{\alpha}+1) = 1.0 \).

C. SECOND MODIFICATION TO THE COMPUTATION PROCEDURE:

METHOD C

In an effort to refine method B so that it might be useful under conditions of very high component reliability and small amounts of testing, a second modification is presented. Method C of computing \( \hat{M} \) and \( \hat{V} \) is an attempt to further smooth the distribution of \( \hat{R}_{s,L(\gamma)} \) and has the effect of adding a partial failure to each component which experiences no failure during testing. For method C computation proceeds as follows:

\[ \hat{M}_C = \frac{k_1}{\mathcal{E}} \sum_{i=1}^{\mathcal{E}} \frac{1}{2} k_1^2 \left( n_i - 1 \right) + \hat{M}_A \]

\[ \hat{V}_C = \frac{k_1}{\mathcal{E}} \sum_{i=1}^{\mathcal{E}} \frac{1}{2} k_1^2 \left( n_i - 1 \right)^2 + \hat{V}_A \]
\[
\hat{\alpha} = \frac{\hat{M}_C^2}{V_C} - 1,
\]

\[
\hat{\beta} = \frac{\hat{M}_C}{V_C}, \text{ and}
\]

\[
\hat{R}_{s,L}(\gamma) = \exp\left[-\frac{\chi^2(\hat{\alpha}+1),1-\gamma}{2\hat{\beta}}\right]
\]

Again, if \(2(\hat{\alpha}+1) < 1.0\), set \(2(\hat{\alpha}+1) = 1.0\).

From these computations it is seen that if every component experiences at least one failure, i.e. \(k_1 = 0\), then method C reduces to method B. On the other hand, if a system consists of say forty components, and there is limited testing relative to the component unreliability of say ten of the components, one might feel that some adjustment should be made to the overall estimate \(\hat{R}_{s,L}(\gamma)\) to account for the lack of testing on those ten components; that is the motivation for this second modification. Furthermore, any method of computing \(\hat{R}_{s,L}(\gamma)\) which yields accurate results with less testing than another method will be more desirable economically.
III. SIMULATION PROCEDURE

The purpose of the computer simulation in this study is to demonstrate the accuracy of the procedures described in section II. The simulation considers the system of k components of Figure 1, where the component reliabilities and hence $R_s$ are assumed to be known. For this system tests are simulated by the Monte Carlo technique, generating failure data which is used to compute $\hat{R}_{s,L(\gamma)}$ by each of the three methods A, B, and C. By replicating the procedure 1000 times, 1000 values of $\hat{R}_{s,L(\gamma)}$ are generated which are then arranged in ascending order. If the method of computing $\hat{R}_{s,L(\gamma)}$ is accurate, then 100(1-$\gamma$)% of the time the true system reliability, $R_s$, will be greater than or equal to $\hat{R}_{s,L(\gamma)}$. Therefore the 100(1-$\gamma$)\textsuperscript{th} percentile point of the 1000 ordered values of $\hat{R}_{s,L(\gamma)}$, $A_{1-\gamma}$, should be equal to or very close to the value of $R_s$ if the method is accurate.

For example, if $\gamma = .10$ then the 90\textsuperscript{th} percentile point or 900\textsuperscript{th} ordered value of $\hat{R}_{s,L(\gamma)}$ should be close to $R_s$. It is this difference, $|R_s - A_{1-\gamma}|$, that is the primary measure of accuracy of the method of estimation. Another measure of the accuracy is the true level of confidence. The true level of confidence is found by determining the percentage of the values of $\hat{R}_{s,L(\gamma)}$ that are less than or equal to $R_s$. If the method is accurate the true level of confidence will be very close to 100(1-$\gamma$)%.
One of the main concerns of this study is to determine what minimum amount of testing is necessary to insure that the method will provide accurate results. This minimum amount of testing will be different for each system and will be dependent on the number of components and their reliabilities. For this reason the quantity

$$TT = \sum_{i=1}^{k} n_i(1-p_i)$$

is used as a measure of the amount of testing relative to the system reliability. Low values of TT occur when combinations of high reliability and small numbers of tests occur, and TT increases directly with the number of tests per component and inversely with component reliability. Intuitively one would feel that as TT decreases the accuracy of the computation procedure will decrease since fewer failures will occur, and less information will be obtained. One purpose of the simulation is to determine for what values of TT the various methods meet desired accuracy goals.

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IV. RESULTS AND CONCLUSIONS

The analysis of accuracy results are shown in Tables I and II. The cases considered are grouped according to the number of components, system reliability, and value of TT. The component reliabilities, $p_i$'s, are held constant in some cases while the number of tests per component, $n_i$'s, are varied, to yield a range of TT values. In other cases the $n_i$'s are fixed and the $p_i$'s varied. Twenty-three cases of systems with reliabilities ranging from approximately .87 to .95 and consisting of 10, 20, and 40 components are considered.

As expected the tendency of all methods to increase in accuracy as TT increases is substantiated. In addition, the tendency of increased accuracy from methods A, to B, to C is also substantiated. Cases 21, 22, and 23 show clearly that for TT = 10.0 the accuracy of the three methods is essentially the same for all three methods of computation, since with this relatively high amount of testing the methods become identical. These three cases also show that as the number of system components changes, if $R_s$ and TT are held constant, the accuracy of the three methods remains essentially unchanged.

It is up to the user to determine what accuracy is acceptable when employing any procedure to compute $\hat{R}_{s,L(\gamma)}$. For illustrative purposes, if one considers $|R_s - A_{1-\gamma}| < .02$ to be acceptable accuracy, then for a 40 component system
with an overall reliability of approximately .90, the testing program should be such that TT = 6.50 is a minimum and TT > 9.0 would be desirable. As TT decreases below 6.50 there is a marked deterioration in accuracy of all three methods as measured by both \( |R_s - A_{\bar{a}}| \) and the true level of confidence versus the desired level of confidence. Another key factor to consider in using any of these methods is that each is sensitive to the individual component test sample size, \( n_i \). If \( n_i \) is quite small, say less than 10, for any component regardless of TT, a failure on one of those \( n_i \) tests will tend to have a disproportionate effect on \( \hat{R}_s, L(\gamma) \). Note that in cases 6 through 10 accuracy proves to be not as good as in cases 1 through 5 for \( k = 20 \) and corresponding values of TT. This is because the \( n_i \) values are generally lower for cases 6 through 10 and in some cases \( n_i \) is less than 10.

The results show little difference in accuracy between the 80% (\( \gamma = .20 \)) and 90% (\( \gamma = .10 \)) confidence levels among the various cases and methods. There is no general tendency for the accuracy to be better at either level although one might expect that accuracy at the 80% confidence level would be better than at the 90% level. For example in case 14A (TT = 6.55) accuracy is better at the 90% level, but in cases 13A (TT = 4.05) and 15A (TT = 9.10) accuracy is better at the 80% level.

These methods are approximations. No analytical justification is given for the modifications suggested. Intuition
has been the guiding factor in development of the methods and the accuracy tests which are based on sound analytical techniques are the basis of acceptance or rejection of the method as a useful tool. Since systems vary widely in their composition, it is not possible to cover them all with a set of specific rules as to when it would be appropriate to use one of the methods for computing \( \hat{R}_{s,L}(\gamma) \). However, from these results two general rules follow:

1. For systems of ten to forty independent components \( TT > 10.0 \) should yield accuracies of \( |R_s - A_{1-\gamma}| < .01 \) provided that,
   
   2. no component is tested less than 10 times, i.e. \( n_i > 10 \) for all components.

If these two rules are followed it can also be concluded that method B should be employed since there should be no appreciable difference in the accuracies of methods A, B, and C, and B is easier to compute than C. Also B does not suffer from the possibility, remote as it would be if \( TT \) were greater than 10.0, of yielding \( \hat{R}_{s,L}(\gamma) = 1.0 \) as would method A. Any system under consideration can be tested using the computer simulation procedure, and as \( TT \) decreases below 10.0, either because of economic or time constraints in the testing program, this would be recommended.
| CASE NO. | K  | \( n_i \) and \( p_i \)                                           | TT  | \( R_s \) | \( \gamma \) | \( A_{1-\gamma} \) | \( |R_s-A_{1-\gamma}| \) | TRUE CONF. LEVEL |
|---------|----|------------------------------------------------------------------|-----|----------|-----------|-----------------|------------------|------------------|
| 1A      | 20 | \( n_i=16, \ i=1, \ldots, 5; \) \( n_i=20, \ i=6, \ldots, 10; \) | 1.45| .9512    | .10       | 1.0000          | .0488            | .770             |
|         |    | \( n_i=40, \ i=11, \ldots, 20; \) \( p_i=.9975, \ i=1, \ldots, 20. \) |     |          | .20       | 1.0000          | .0488            | .563             |
| 1B      | 20 | "                                                                | 1.45| .9512    | .10       | .9659           | .0147            | NC*              |
| 1C      | 20 | "                                                                | 1.45| .9512    | .10       | .9402           | .0110            | 1.000            |
| 2 A     | 20 | \( n_i=6, \ i=1, \ldots, 5; \) \( n_i=30, \ i=6, \ldots, 10; \) | 2.95| .9512    | .10       | .9770           | .0258            | .668             |
|         |    | \( n_i=100, \ i=11, \ldots, 20. \) \( p_i=.9975, \ i=1, \ldots, 20. \) |     |          | .20       | .9702           | .0190            | .514             |
| 2 B     | 20 | "                                                                | 2.95| .9512    | .10       | NC              | NC               | NC               |
| 2 C     | 20 | "                                                                | 2.95| .9512    | .10       | .9566           | .0054            | .869             |

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case

**TABLE I.**
<table>
<thead>
<tr>
<th>CASE NO.</th>
<th>K</th>
<th>( n_i ) and ( p_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3A</td>
<td>20</td>
<td>( n_i=14, i=1, \ldots, 5; n_i=60, i=6, \ldots, 10; n_i=125, i=1, \ldots, 20; p_i=0.9975, i=1, \ldots, 20. )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TT</td>
</tr>
<tr>
<td>3B</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3C</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4A</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4B</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4C</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case

TABLE I.
| CASE NO. | K | \( n_i \) and \( p_i \) | \( \alpha \) | \( A_1-\alpha \) | \( |R_s-A_1-\alpha| \) | TRUE CONF. LEVEL |
|--------|---|-----------------|--------|-------------|----------------|-------------|
| 5A     | 20| \( n_i=28 \), \( i=1,\ldots,5; \) \( n_i=100 \), \( i=6,\ldots,10; \) \( n_i=300 \), \( i=11,\ldots,20; \) \( p_i=.9975 \), \( i=1,\ldots,20. \) | .10   | .9650       | .0138           | .729       |
|        |    |                 | .20   | .9627       | .0113           | .622       |
| 5B     | 20| \( \alpha \)     | .10   | NC          | NC              | NC          |
|        |    |                 | .20   | NC          | NC              | NC          |
| 5C     | 20| \( \alpha \)     | .10   | .9635       | .0123           | .730       |
|        |    |                 | .20   | .9671       | .0159           | .673       |
| 6A     | 20| \( n_i=8 \), \( i=1,\ldots,5; \) \( n_i=10 \), \( i=6,\ldots,10; \) \( n_i=20 \), \( i=11,\ldots,20; \) \( p_i=.995 \), \( i=1,\ldots,20. \) | .10   | 1.0000      | .0954           | .773       |
|        |    |                 | .20   | 1.0000      | .0954           | .521       |
| 6B     | 20| \( \alpha \)     | .10   | .9312       | .0266           | NC          |
|        |    |                 | .20   | .9578       | .0532           | NC          |
| 6C     | 20| \( \alpha \)     | .10   | .8808       | .0238           | NC          |
|        |    |                 | .20   | .9163       | .0117           | NC          |

*NC indicates No Change from corresponding result of previous method.*

Summary of Accuracy Results by Case

TABLE I.
| CASE NO. | K  | \(n_i\) and \(p_i\) | TT | \(R_s\) | \(\gamma\) | \(A_{1-\gamma}\) | \(|R_s-A_{1-\gamma}|\) | TRUE CONF. LEVEL |
|---------|----|---------------------|----|---------|---------|----------------|----------------|-----------------|
| 7A      | 20 | \(n_i=3, i=1,\ldots,5;\)  
\(n_i=15, i=6,\ldots,10;\)  
\(n_i=50, i=11,\ldots,20;\)  
\(p_i=.995, i=1,\ldots,20.\) | 2.95 | .9046 | .10 | .9541 | .0495 | .638 |
| 7B      | 20 | " | 2.95 | .9046 | .10 | NC | NC | NC |
| 7C      | 20 | " | 2.95 | .9046 | .10 | .8946 | .0100 | NC |
| 8 A     | 20 | \(n_i=7, i=1,\ldots,5;\)  
\(n_i=35, i=6,\ldots,10;\)  
\(n_i=60, i=11,\ldots,20;\)  
\(p_i=.995, i=1,\ldots,20.\) | 4.05 | .9046 | .10 | .9362 | .0316 | .697 |
| 8 B     | 20 | " | 4.05 | .9046 | .10 | NC | NC | NC |
| 8 C     | 20 | " | 4.05 | .9046 | .10 | .9295 | .0249 | .713 |

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case

TABLE I.
| CASE NO. | K | $n_i$ and $p_i$ | TT | $R_s$ | $\gamma$ | $A_1-\gamma$ | $|R_s-A_1-\gamma|$ | TRUE CONF LEVEL |
|----------|---|----------------|----|------|--------|-------------|----------------|-----------------|
| 9A       | 20| $n_i=12$, $i=1,\ldots,5$; $n_i=50$, $i=6,\ldots,10$; $n_i=100$, $i=11,\ldots,20$; $p_i=.995$, $i=1,\ldots,20$. | 6.55 | .9046 | .10 | .9346 | .0300 | .711 |
| 9B       | 20| " | 6.55 | .9046 | .10 | NC | NC | NC |
| 9C       | 20| " | 6.55 | .9046 | .10 | .9312 | .0266 | .749 |
| 10A      | 20| $n_i=14$, $i=1,\ldots,5$; $n_i=50$, $i=6,\ldots,10$; $n_i=150$, $i=11,\ldots,20$; $p_i=.995$, $i=1,\ldots,20$. | 9.10 | .9046 | .10 | .9315 | .0269 | .723 |
| 10B      | 20| " | 9.10 | .9046 | .10 | NC | NC | NC |
| 10C      | 20| " | 9.10 | .9046 | .10 | .9283 | .0237 | .726 |

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case TABLE I.
| CASE NO. | K  | n_i and p_i                        | TT  | R_s  | γ   | A_1-γ | |R_s-A_1-γ| TRUE CONF. LEVEL |
|---------|----|-----------------------------------|-----|------|-----|--------|----------------|------------------|
| 11A     | 40 | n_i=5, i=1,...,15; n_i=10, i=16,...,30; n_i=50, i=31,...,40; p_i=.998, i=1,...,40. | 1.45 | .9230 | .10 | 1.0000 | .0770 | .419           |
| 11B     | 40 |                                   | 1.45 | .9230 | .10 | 0.9727 | .0497 | NC             |
| 11C     | 40 |                                   | 1.45 | .9230 | .10 | 0.9450 | .0220 | .772           |
| 12A     | 40 | n_i=5, i=1,...,15; n_i=10, i=16,...,30; n_i=100, i=31,...,35; n_i=150, i=36,...,40; p_i=.998, i=1,...,40. | 2.95 | .9230 | .10 | 0.9847 | .0617 | .404           |
| 12B     | 40 |                                   | 2.95 | .9230 | .10 | NC      | NC    | NC             |
| 12C     | 40 |                                   | 2.95 | .9230 | .10 | 0.9539 | .0309 | .495           |

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case TABLE I.
| CASE NO. | K | \( n_1 \) and \( p_i \) | TT | \( R_S \) | \( \gamma \) | \( A_1 - \gamma \) | \( |R_S - A_1 - \gamma| \) | TRUE CONFL. LEVEL |
|---------|---|---------------------|----|--------|--------|----------------|----------------|----------------|
| 13A     | 40 | \( n_i = 5, i = 1, ... 5; n_i = 10, i = 6, ... 10; \) \( n_i = 20, i = 11, ... 20; n_i = 50, i = 21, ... 30; \) \( n_i = 100, i = 31, ... 35; n_i = 150, i = 36, ... 40; \) \( p_i = 0.998, i = 1, ... 40. \) | 4.05 | .9230 | .10 | .9650 | .0420 | .649 |
| 13B     | 40 | \( \ldots \) | 4.05 | .9230 | .10 | NC | NC | NC |
| 13C     | 40 | \( \ldots \) | 4.05 | .9230 | .10 | .9524 | .0294 | .701 |
| 14A     | 40 | \( n_i = 5, i = 1, ... 5; n_i = 50, i = 6, ... 20; \) \( n_i = 100, i = 21, ... 30; \) \( n_i = 150, i = 31, ... 40; \) \( p_i = 0.998, i = 1, ... 40. \) | 6.55 | .9230 | .10 | .9396 | .0166 | .785 |
| 14B     | 40 | \( \ldots \) | 6.55 | .9230 | .10 | NC | NC | NC |
| 14C     | 40 | \( \ldots \) | 6.55 | .9230 | .10 | .9356 | .0126 | .825 |

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case TABLE I.
| CASE NO. | K | \( n_i \) and \( p_i \) | TT | \( R_s \) | \( \gamma \) | \( A_{1-\gamma} \) | \( |R_s-A_{1-\gamma}| \) | TRUE CONF. LEVEL |
|---------|---|----------------|----|-------|------|----------|----------------|-----------------|
| 15A     | 40| \( n_i = 50, i=1,\ldots,10; \)  
\( n_i = 100, i=11,\ldots,20; \)  
\( n_i = 150, i=21,\ldots,40; \)  
\( p_i = .999, i=1,\ldots,40. \) | 9.00 | .9230 | .10 | .9294 | .0064 | .861 |
| 15B     | 40| " | 9.00 | .9230 | .10 | NC | NC | NC |
| 15C     | 40| " | 9.00 | .9230 | .10 | .9292 | .0062 | .863 |
| 16A     | 40| \( n_i = 15, i=1,\ldots,40 \)  
\( p_i = .999, i=1,\ldots,5; p_i = .998, i=6,\ldots,20; \)  
\( p_i = .997, i=21,\ldots,35; \)  
\( p_i = .990, i=36,\ldots,40. \) | 1.95 | .8778 | .10 | 1.0000 | .1222 | .851 |
| 16B     | 40| " | 1.95 | .8778 | .10 | .9078 | .0300 | NC |
| 16C     | 40| " | 1.95 | .8778 | .10 | NC | NC | NC |

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case

TABLE I.
| CASE NO. | K | \( n_i \) and \( p_i \) | TT | \( R_s \) | \( \gamma \) | \( A_{1-\gamma} \) | \( |R_s-A_{1-\gamma}| \) | TRUE CONE LEVEL |
|---------|---|-----------------|----|---------|------|-------|-----------------|-----------------|
| 17 A    | 40 | \( n_i=20, \ i=1, \ldots, 40 \); \( p_i=.999, i=1, \ldots, 5; p_i=.998, i=6, \ldots, 20 \); \( p_i=.997, i=21, \ldots, 35 \); \( p_i=.990, i=36, \ldots, 40 \). | 2.60 | 0.8778 | 0.10 | 0.8858 | 0.0080 | 0.746 |
| 17 B    | 40 | " | 2.60 | 0.8778 | 0.10 | NC | NC | NC |
| 17 C    | 40 | " | 2.60 | 0.8778 | 0.20 | 0.9188 | 0.0410 | 0.732 |
| 18 A    | 40 | \( n_i=30, \ i=1, \ldots, 40 \); \( p_i=.999, i=1, \ldots, 5; p_i=.998, i=6, \ldots, 20 \); \( p_i=.997, i=21, \ldots, 35 \); \( p_i=.990, i=36, \ldots, 40 \). | 3.90 | 0.8778 | 0.10 | 0.9236 | 0.0458 | 0.897 |
| 18 B    | 40 | " | 3.90 | 0.8778 | 0.20 | 0.9017 | 0.0239 | 0.746 |
| 18 C    | 40 | " | 3.90 | 0.8778 | 0.10 | 0.9229 | 0.0451 | NC |

*NC indicates No Change from corresponding result of previous method.*

Summary of Accuracy Results by Case

TABLE I.
| CASE NO. | K | n_i and p_i                                                                 | TT | R_s  | \( \gamma \) | A_{1-\gamma} | \( |R_s-A_{1-\gamma}| \) | TRUE CONF. LEVEL |
|---------|---|-----------------------------------------------------------------------------|----|------|-------------|--------------|----------------|-----------------|
| 19 A    | 40 | \( n_i = 50, \ i = 1, \ldots, 40 \)                                        | 6.50 | .8778 | .10         | .8975        | .0197          | 884             |
|         |    | \( p_i = .999, \ i = 1, \ldots, 5 \); \( p_i = .998, \ i = 6, \ldots, 20; \) |    |      |             |              |                |                 |
|         |    | \( p_i = .997, \ i = 21, \ldots, 35; \)                                    |    |      |             |              |                |                 |
|         |    | \( p_i = .990, \ i = 36, \ldots, 40. \)                                    |    |      |             |              |                |                 |
| 19 B    | 40 |                                                                             | 6.50 | .8778 | .10         | NC           | NC             | NC              |
| 19 C    | 40 |                                                                             | 6.50 | .8778 | .10         | .8971        | .0193          | NC              |
| 20 A    | 40 | \( n_i = 70, \ i = 1, \ldots, 40 \)                                        | 9.10 | .8778 | .10         | .8745        | .0033          | 902             |
|         |    | \( p_i = .999, \ i = 1, \ldots, 5 \); \( p_i = .998, \ i = 6, \ldots, 20; \) |    |      |             |              |                |                 |
|         |    | \( p_i = .997, \ i = 21, \ldots, 35; \)                                    |    |      |             |              |                |                 |
|         |    | \( p_i = .990, \ i = 36, \ldots, 40. \)                                    |    |      |             |              |                |                 |
| 20 B    | 40 |                                                                             | 9.10 | .8778 | .10         | NC           | NC             | NC              |
| 20 C    | 40 |                                                                             | 9.10 | .8778 | .10         | .8743        | .0035          | NC              |

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case

**TABLE I.**
| CASE NO. | K  | $n_i$ and $p_i$                                                                 | TT  | $R_s$ | $\gamma$ | $A_{1-\gamma}$ | $|R_s-A_{1-\gamma}|$ | TRUE CONF LEVEL |
|---------|----|---------------------------------------------------------------------------------|-----|-------|----------|----------------|-----------------|----------------|
| 21A     | 10 | $n_i=125, i=1,\ldots,10;$ $p_i=.992, i=1,\ldots,10.$                          | 10.0| .9228 | .10      | .9279         | .0051           | .879           |
| 21B     | 10 | "                                                                               | 10.0| .9228 | .10      | NC            | NC             | NC             |
| 21C     | 10 | "                                                                               | 10.0| .9228 | .10      | .9270         | .0042           | NC             |
| 22A     | 20 | $n_i=125, i=1,\ldots,20;$ $p_i=.996, i=1,\ldots,20.$                          | 10.0| .9229 | .10      | .9281         | .0052           | .869           |
| 22B     | 20 | "                                                                               | 10.0| .9229 | .10      | NC            | NC             | NC             |
| 22C     | 20 | "                                                                               | 10.0| .9229 | .10      | .9278         | .0048           | NC             |

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case

**TABLE I**
### TABLE I.
Summary of Accuracy Results by Case

<table>
<thead>
<tr>
<th>CASE</th>
<th>n and p_i</th>
<th>Rs</th>
<th>A1-γ</th>
<th>δ</th>
<th>TT</th>
<th>TRUE LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>n=125, p_i = .998, i=1,...,40; 23 40</td>
<td>10.0</td>
<td>.9230</td>
<td>10</td>
<td>10</td>
<td>10.0</td>
</tr>
<tr>
<td>B</td>
<td>n=125, p_i = .998, i=1,...,40; 23 40</td>
<td>10.0</td>
<td>.9230</td>
<td>10</td>
<td>10</td>
<td>10.0</td>
</tr>
</tbody>
</table>

*NC indicates No Change from corresponding result of previous method.*
| Case No. | k | TT | \( R_s \) | \( |R_s - A_{1-\gamma}| \) |
|---------|---|----|---------|-----------------|
|         |   |    |         | A   | B   | C |
| 16      | 40| 1.95| .8778   | .1222| .0300| NC |
| 6       | 20| 1.45| .9046   | .0954| .0266| .0238 |
| 11      | 40| 1.45| .9230   | .0770| .0497| .0220 |
| 1       | 20| 1.45| .9512   | .0488| .0147| .0110 |
| 17      | 40| 2.60| .8778   | .0080| NC   | .0070 |
| 7       | 20| 2.95| .9046   | .0495| NC   | .0100 |
| 12      | 40| 2.95| .9230   | .0617| NC   | .0309 |
| 2       | 20| 2.95| .9512   | .0258| NC   | .0054 |
| 18      | 40| 3.90| .8778   | .0458| NC   | .0451 |
| 8       | 20| 4.05| .9046   | .0316| NC   | .0249 |
| 13      | 40| 4.05| .9230   | .0420| NC   | .0294 |
| 3       | 20| 4.05| .9512   | .0179| NC   | .0147 |
| 19      | 40| 6.50| .8778   | .0197| NC   | .0193 |
| 9       | 20| 6.55| .9046   | .0300| NC   | .0266 |
| 14      | 40| 6.55| .9230   | .0166| NC   | .0126 |
| 4       | 20| 6.55| .9512   | .0157| NC   | .0115 |

Summary of Accuracy Results for \( \gamma = .10 \)

Table II
| Case No. | k | TT | R_s | $|R_s - A_{1-\gamma}|$ |
|---------|---|----|-----|-----------------|
|         |   |    |     | A   | B   | C   |
| 20      | 40| 9.10| .8778| .0033 | NC  | .0035 |
| 10      | 20| 9.10| .9046| .0269 | NC  | .0237 |
| 15      | 40| 9.00| .9230| .0064 | NC  | .0062 |
| 5       | 20| 9.10| .9512| .0138 | NC  | .0123 |
| 21      | 10| 10.0| .9228| .0051 | NC  | .0042 |
| 22      | 20| 10.0| .9229| .0052 | NC  | .0048 |
| 23      | 40| 10.0| .9230| .0051 | NC  | .0050 |

Summary of Accuracy Results for $\gamma = .10$

Table II
| Case No. | k  | TT   | Rs   | $|R_s - A_1 - \gamma|$ |
|---------|----|------|------|-----------------|
|         |    |      |      | A               | B               | C               |
| 16      | 40 | 1.95 | .8778| .0136           | NC              | .0124           |
| 6       | 20 | 1.45 | .9046| .0954           | .0532           | .0117           |
| 11      | 40 | 1.45 | .9230| .0447           | NC              | .0414           |
| 1       | 20 | 1.45 | .9512| .0488           | .0280           | .0088           |
| 17      | 40 | 2.60 | .8778| .0080           | NC              | .0070           |
| 7       | 20 | 2.95 | .9046| .0360           | NC              | .0243           |
| 12      | 40 | 2.95 | .9230| .0571           | NC              | .0420           |
| 2       | 20 | 2.95 | .9512| .0190           | NC              | .0138           |
| 18      | 40 | 3.90 | .8778| .0239           | NC              | .0234           |
| 8       | 20 | 4.05 | .9046| .0278           | NC              | .0213           |
| 13      | 40 | 4.05 | .9230| .0318           | NC              | .0295           |
| 3       | 20 | 4.05 | .9512| .0148           | NC              | .0129           |
| 19      | 40 | 6.50 | .8778| .0055           | NC              | .0059           |
| 9       | 20 | 6.55 | .9046| .0246           | NC              | .0229           |
| 14      | 40 | 6.55 | .9230| .0198           | NC              | .0151           |
| 4       | 20 | 6.55 | .9512| .0096           | NC              | .0088           |

Summary of Accuracy Results for $\gamma = .20$

Table II
| Case No. | k  | TT  | \( R_s \) | \(| R_s - A_{1-\gamma} | \) |
|---------|----|------|-----------|----------------|
|         |    |      | A | B | C |
| 20      | 40 | 9.10 | .8778 | .0014 | NC | .0016 |
| 10      | 20 | 9.10 | .9046 | .0215 | NC | .0181 |
| 15      | 40 | 9.00 | .9230 | .0052 | NC | .0050 |
| 5       | 20 | 9.10 | .9512 | .0115 | NC | .0159 |
| 21      | 10 | 10.0 | .9228 | .0062 | NC | .0052 |
| 22      | 20 | 10.0 | .9229 | .0061 | NC | .0052 |
| 23      | 40 | 10.0 | .9230 | .0062 | NC | .0060 |

Summary of Accuracy Results for \( \gamma = .20 \)

Table II
SIMULATION PROGRAM TO EVALUATE THE ACCURACY OF THE LOWER CONFIDENCE ESTIMATE FOR SERIES SYSTEM RELIABILITY

COMPO = METHOD A OF TEXT
COMP1 = METHOD B OF TEXT
COMP2 = METHOD C OF TEXT

DIMENSION N(50), P(50), VS(50), GAMMA(5)
DIMENSION NT(50), NST(50), NFT(50)
DIMENSION A(5), CONF(5), RSLCL(1000)
DIMENSION MF(50)
DIMENSION PCNTIL(5), CSTABL(100, 2)

DATA N/50*0.0/, P/50*0.0/, VS/50*0.0/, GAMMA/5*0.0/
DATA A/5*0.0/, CONF/5*0.0/, RSLCL/1000*0.0/
DATA NT/50*0.0/, NST/50*0.0/, NFT/50*0.0/
DATA MF/50*0.0/
DATA PCNTIL/5*0.0/, CSTABL/250*0.0/

COUNT = 0
ICOUNT = 0
NFI = 0
ISTART = 65549
IRN = ISTART
M = 12492
KK = 8 * M + 3
ISTOP = 0
IFLAGJ = 0
IFLAG1 = 0
IFLAG2 = 1
IF (IFLAGJ.EQ.0) GO TO 0020
JCOMP1 = 0
JCOMP2 = 0
IFLAG2 = 0
GO TO 0050

0020 IF (IFLAG1.EQ.0) GO TO 0030
JCOMP1 = 1
JCOMP2 = 0
GO TO 0050

0030 IF (IFLAG2.EQ.0) GO TO 0045
JCOMP1 = 0
JCOMP2 = 1
ISTOP = 1
GO TO 0050

0045 WRITE (6,0046)
0046 FORMAT (12X, 'IFLAG VALUES SCREWED UP!')

STOP

READ INPUT DATA
0050 READ (5, 0100) K, NGAMMA, NREP
0100 FORMAT (315)
C100 READ (5,0200) (GAMMA(J), J=1, NGAMMA)
0200 FORMAT (5F10.5)
0300 ILOOP = 0
0400 FORMAT (5,0400) N(ILOOP), P(ILOOP)
0400 FORMAT (115,1F10.5)
0500 CONTINUE
0700 ILOOP = ILOOP + 1
0700 PCNTIL(ILOOP) = 1.0 - GAMMA(ILOOP)
0800 IF(ILOOP.LT.K) 50 TO 0300
0900 DO 0500 I = 1, 100
0900 READ (5,0500) (CSTABL(I,J), J=1,2)
0900 CONTINUE
0900 FORMAT (2F10.5)
0900 WRITE (6,0780) NREP
0900 FORMAT (1X,10, 'ERROR, NREP NOT EQUAL TO 1000 OR 100, NREP=', 10)
0900 STOP
0900 FORMAT (6,0800) K, NGAMMA, NREP
0900 WRITE (6,0800) K, NGAMMA, NREP
0900 FORMAT (7X, 'INPUT VALUES FOR SIMULATION', 10)
0900 IF(K.EQ.1) 20, 1680, 1
0900 format ('1', 5X, 'NUMBER OF COMPONENTS IN SERIES', 10)
0900 format ('1', 5X, 'NUMBER OF CONFIDENCE LEVELS', 10)
0900 format ('1', 5X, 'NUMBER OF REPLICATIONS', 10)
0900 IF(K.EQ.2) 110, 1680, 2
0900 format ('2', 12X, 'COMPONENT', 5X, 10)
0900 format ('2', 12X, 'NUMBER OF', 7X, 10)
0900 format ('2', 12X, 'RELIABILITY TEST TRIALS TEST FAILURES', 10)
0900 IF(K.EQ.3) 110, 1680, 3
0900 format ('3', 12X, 'COMPONENT', 5X, 10)
0900 format ('3', 12X, 'NUMBER OF', 7X, 10)
0900 format ('3', 12X, 'CHI-SQUARED DISTRIBUTION', 10)
0900 format ('3', 12X, 'PERCENTILE', 1X, 5X, 4X, 10)
0900 IF(K.EQ.4) 110, 1680, 4
0900 format ('4', 12X, 'COMPONENT', 5X, 10)
0900 format ('4', 12X, 'NUMBER OF', 7X, 10)
0900 format ('4', 12X, 'CHI-SQUARED DISTRIBUTION', 10)
0900 format ('4', 12X, 'PERCENTILE', 1X, 5X, 4X, 10)
0900 IF(K.EQ.5) 110, 1680, 5
0900 format ('5', 12X, 'COMPONENT', 5X, 10)
0900 format ('5', 12X, 'NUMBER OF', 7X, 10)
0900 format ('5', 12X, 'CHI-SQUARED DISTRIBUTION', 10)
0900 format ('5', 12X, 'PERCENTILE', 1X, 5X, 4X, 10)
0900 IF(K.EQ.6) 110, 1680, 6
0900 format ('6', 12X, 'COMPONENT', 5X, 10)
0900 format ('6', 12X, 'NUMBER OF', 7X, 10)
0900 format ('6', 12X, 'CHI-SQUARED DISTRIBUTION', 10)
0900 format ('6', 12X, 'PERCENTILE', 1X, 5X, 4X, 10)
1000 CONTINUE
1000 WRITE (6,1000) ILOOP, GAMMA(ILOOP)
1100 FORMAT (1X, '1', 12X, 'COMPONENT', 5X, 10)
1100 format ('1', 12X, 'NUMBER OF', 7X, 10)
1100 format ('1', 12X, 'RELIABILITY TEST TRIALS TEST FAILURES', 10)
1200 ILOOP = ILOOP + 1
1200 WRITE (6,1200) ILOOP, P(ILOOP), N(ILOOP), NS(ILOOP), NF(ILOOP)
1300 FORMAT (12X, '1', 12X, 'COMPONENT', 5X, 10)
1300 format ('1', 12X, 'NUMBER OF', 7X, 10)
1300 format ('1', 12X, 'CHI-SQUARED DISTRIBUTION', 10)
1300 format ('1', 12X, 'PERCENTILE', 1X, 5X, 4X, 10)
1400 CONTINUE
1500 FORMAT (15X, '1', (CSTABL(I,J), J=1,2)
1500 WRITE (6,1500) I, (CSTABL(I,J), J=1,2)
1600 CONTINUE
1600 CONTINUE
1700 CONTINUE
1700 CONTINUE
1800 CONTINUE
1800 CONTINUE
1900 CONTINUE
1900 CONTINUE
2000 CONTINUE
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2100 CONTINUE
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9700 CONTINUE
9700 CONTINUE
9800 CONTINUE
9800 CONTINUE
9900 CONTINUE
9900 CONTINUE
K1 = K - 1
I = 1

1610 I = I + 1
IF (M(I), GE, N(I)) GO TO 1620
NMAX = N(I)
GO TO 1630
1620 NMAX = N(I)
1630 IF (I, GE, K1) GO TO 1640
I = I + 1
GO TO 1610
1640 NMAX1 = NMAX - 1
XNMAK1 = NMAX1

COMPUTE SERIES SYSTEM RELIABILITY BY PRODUCT ASSUMPTION
RCOMP = 1.0
ILOOP = 0
1700 ILOOP = ILOOP + 1
RCOMP = RCOMP * P(ILOOP)
IF (ILOOP, LT, K) GO TO 1700
WRITE (6, 1800)
1800 FORMAT ('I', 7X, 'SIMULATION OUTPUT', ' //

COMPUTE TT
TT = 0.0
DO 1900 I = 1, K
T = M(I) * (1.0 - P(I))
TT = TT + T
1900 CONTINUE

BEGIN SIMULATION
1950 L = 1
2300 G = GAMMA(L)
J = 1

GENERATE K VALUES OF NS(I) BY SIMULATING TESTS
2100 I = 1
I! = 1
2200 IRN = IRN * K
RN = 0.5 + FLOAT(IRN) * 2.328306E-10
KOUNT = KOUNT + 1
IF (RN, GT, P(I)) GO TO 2300
NS(I) = NS(I) + 1
GO TO 2400
2300 ME(I) = ME(I) + 1
2400 II = II + 1
IF (II, LE, N(I)) GO TO 2200
II = 1
I = 1 + 1
IF (I, LE, K) GO TO 2200

SIMULATION COMPLETE FOR REPLICATION J
COMPUTE W, V, ALPHA, AND BETA
I = 1
EXPU = 0.0
VARU = 0.0
2500 INC = 0
2600 IF (NF(I), EQ., 0) GO TO 2700
2600 S = VS(I) + INC
2600 EXPU = EXPU + 1.0/S
2600 VARU = VARU + 1.3 * (S + S)
2600 INC = INC + 1
2700 IF (NF(I), GT, INC) GO TO 2600
2700 I = I + 1
2700 IF (I, LE, K) GO TO 2500
2700 IF (COMP1, EQ., 0) GO TO 2730

COMPUTATION 2

SUM2 = 0.0
SUM3 = 0.0
SUM4 = 0.0
2720 DC 2720 I = 1,
2720 IF (NF(I), NE, 0) GO TO 2720
2720 XNS1 = N(I) - 1
2720 SUM2 = SUM2 + 1.0/XNS1
2720 SUM3 = SUM3 + 1.3/(XNS1 * XNS1)
2720 SUM4 = SUM4 + 1.0
2720 CONTINUE
2730 D1 = 2.0 * SUM4 * SUM4
2730 IF (D1, EQ, 0.0) GO TO 2730
2730 EXPU = EXPU + SUM2/D1
2730 VARU = VARU + SUM3/D1
2730 IF (VARU, GT, 0.3) GO TO 2800
2730 IF (COMP2, EQ., 0) GO TO 2750

COMPUTATION 3

NF1 = NF1 + 1
EXPU = 1.0 / XMAX1
VARU = VARU * EXPU
BETA = XMAX1
ALPHA = 0.0
2750 IF (BETA, EQ., 2.0 * BETA) GO TO 3200

COMP WITH NO FAILURES

2750 BETA = 0.0
ALPHA = -1.0
2750 D1 = 1.0
CS = 0.0
2750 RSLC(I) = 1.0
2750 GO TO 3200

COMP WITH NO FAILURES

2300 BETA = EXPU / VARU
2300 TOBETA = 2.0 * BETA
2300 ALPHA = BETA * EXPU - 1.0
DF = TDBETA * EXPU
IF (DF.LT.100.) GO TO 2870
WRITE (6, 2860) J, DF
2860 FORMAT (1X, 'IN REP', 15, ', DF GT 100, DF=', F10.5, ')
STOP
2870 IF (DF.GE.1.) GO TO 3200
WRITE (6, 2870) J, DF
2880 FORMAT (1X, 'INREP', 15, ', DF LT 1.), DF=', F10.5,
/ 100 SET TO 1.0)
DF = 1.0
SEARCH CHI SQUARE TABLE FOR CS
3200 NDF = DF
NDF1 = NDF + 1
CS = (CSTABL(NDF1,L) - CSTABL(NDF,L)) * (DF-NDF) + CSTABL(NDF,L)
COMPUTE THE ESTIMATED SYSTEM RELIABILITY LCL RESULTING
FROM THIS SEQUENCE OF SIMULATED TESTS
PSLCL(J) = 1.0 / (2.7183 ** (CS/TDBETA))
CASELCL(J), L, 2.1) GO TO 3310
IF (DF.LT.1.0.) DF = 1.0
3300 IF (IOUT.EQ.0.) GO TO 3550
OUTPUT FROM REP ETITION
3310 WRITE (6, 3320) J
3320 FORMAT (1X, 'RESULTS OF REPLICATION', 15, ')
WRITE (6, 3110)
3330 ILOOP = ILOOP + 1
WRITE (6, 1300) ILOOP, P(ILOOP), N(ILOOP), NS(ILOOP), NF(ILOOP)
IF (ILOOP.LT.K) GO TO 3330
WRITE (6, 3340) EXPU, VARI, ALPHA, BETA
3340 FORMAT (1X, 'M=', F10.5, ',', 16X, 'V=', F10.5, ',', 16X, 'BETA=', F10.5, 
/ / *)
WRITE (6, 3400)
3400 FORMAT (1X, 'RESULTS USING BRAN APPROXIMATION', ')/
WRITE (6, 3500) DF, CS, PCNTIL(L), RSLCL(J)
3500 FORMAT (1X, 'CHI SQUARE=', F10.5, ',', 16X, 'ESTIMATED', '
/ *', F4.2, 'LCL FOR SYSTEM RELIABILITY=', F10.5, '/)
2550 IF (IPOSI.EQ.0.) GO TO 4100
ZERO OUT NS(I), NF(I), AND ADD N(I) TO NT(I)
4100 IF (I = 1) THEN
4200 CONTINUE
4100 CONTINUE
4200 CONTINUE
4210 IF (J.LE.INREP) GO TO 2100
PRINT SUMMARY OF RANDOM NUMBER GENERATION AND TEST RESULTS

WRITE (6, 4300)
4300 FORMAT ('II', 17X, 'SUMMARY OF SIMULATION RESULTS', //)
4400 FORMAT (1X, //', 12X, 'NUMBER OF RANDOM NUMBERS GENERATED=', //, 11I1), //', 12X, 'RANDOM NUMBER GENERATOR SEED=', //, I10, //, 212X, 'LAST RANDOM NUMBER GENERATOR SEED=', //, I15, //)
WRITE (6, 1100)
ILOOP = 0
4500 ILOOP = ILOOP + 1
WRITE (6, 1300) ILOOP, NT(ILOOP), NST(ILOOP), NFT(ILOOP)
IF (ILOOP.LT.4) GO TO 4500
WRITE (6, 4600) NT(ILOOP.NFRT)
4550 FORMAT (//'12X, 'THERE WERE ', 15, ' REPLICATIONS WITH NO FAILURES //'
1, //)
IF (IOUT.EQ.0) GO TO 5200
WRITE (6, 4600)
4600 FORMAT (1X, //', 12X, 'UNORDERED VALUES OF SYSTEM', //, 11X, 'RELIABILITY LCLS, READ LEFT TO RIGHT', //)
4800 KOUNT2 = 10
KOUNT3 = 1
4850 WRITE(6, 4900) (RSLCL(J), J = KOUNT3, KOUNT2)
4900 FORMAT(1X, 'BRAM', 7X, 10F10.5)
5100 IF (KOUNT2.EQ.NREP) GO TO 5200
KOUNT2 = KOUNT2 + 10
KOUNT3 = KOUNT3 + 10
GO TO 4850
ARRANGE THE SYSTEM RELIABILITY LCL VALUES IN ASCENDING ORDER
5200 NP1 = NREP - 1
DO 5400 JJ = 1, NR1
KR = NREP - JJ
KOUNT1 = 0
DO 5300 II = 1, KS
IF (RSLCL(II).LE.RSLCL(II + 1)) GO TO 5300
KOUNT1 = KOUNT1 + 1
TEMP = RSLCL(II)
RSLCL(II) = RSLCL(II + 1)
RSLCL(II + 1) = TEMP
5300 CONTINUE
IF (KOUNT1.EQ.J) GO TO 5450
5400 CONTINUE
FIND THE VALUE OF RSLCL WHICH IS THE (1-G)TH PERCENTILE
OF ALL THE ORDERED VALUES
5500 XNREP = NREP
JI = PCNTIL(L) * XNREP
RSLCL1 = RSLCL(JI)
FIND HOW MANY VALUES OF RSLCL(J) ARE .LE. TO RCOMP
FROM THIS FIND THE TRUE LEVEL OF CONFIDENCE
DO 5600 JJ = 1, NREP
IF (RSCL(JJ).LT.RCOMP) GO TO 5600
A(L) = JJ-1
GO TO 5700
5600 CONTINUE
5700 CONF(L) = A(L) / XNREP
PRINT 5800
WRITE (6,5800)
5800 FORMAT (1X,12X,'ORDERED VALUES OF SYSTEM RELIABILITY',
1     'LCLS, READ LEFT TO RIGHT',/)
5820 KOUNT2 = 10
KOUNT3 = 1
5830 WRITE (6,4900) (RSCL(J), J=KOUNT3, KOUNT2)
IF(SIXPOI.EQ.0) GO TO 5840
WRITE (6,5300) (RSLOPT(J), J=KOUNT3, KOUNT2)
5840 IF(KOUNT2.EQ.XNREP) GO TO 5850
KOUNT2 = KOUNT2 + 10
KOUNT3 = KOUNT3 + 10
GO TO 5830
5850 WRITE (6,3400)
WRITE (6,5900) RCOMP
5900 FORMAT (1X,12X,'SYSTEM RELIABILITY BY PRODUCT',
1     'ASSUMPTION RULES',F10.5,/)  
WRITE (6,6000) PCNTIL(L),RSCL1
6000 FORMAT (12X,'THE',F5.2,'PERCENTILE OF THE ORDERED VALUES',
1     'OF SYSTEM RELIABILITY LCL=',F10.5,/) 
WRITE (6,6100) ALL,CONF(L)
6100 FORMAT (12X,'THERE WERE ',F6.2,'VALUES OF RSCL1',
1     'LESS THAN OR EQUAL TO RCOMP',12X,'LCL',
2     'THE TRUE LEVEL OF CONFIDENCE IS',F10.5)
WRITE (6,6200)
6200 FORMAT (12X,'TT=',F10.3)
6260 IF(L.GE.MNGAMMA) GO TO 6300
NE1 = 0
L = L + 1
GO TO 6200
6300 IF (IFLAG2.NE.0) GO TO 6400
PRESERVE FOR COMP1
IRN = ISTART
NE1 = 0
ICOMP1 = 1
ICOMP2 = 0
IFLAG2 = 1
GO TO 6500
6400 IF (ISTARNE .GT. 0) GO TO 6700
PRESERVE FOR COMP2
IRN = ISTART
ICOMP1 = 0
ICOMP2 = 1
NFI = 0
ISTOP = 1

ZED OUT TOTALS
6500 DO 6600 I = 1, K
NST(I) = 0
NFT(I) = 0
6600 CONTINUE
GO TO 1650
6700 STOP
END

INPUT DATA CARDS INCLUDE VALUES FOR K, NO. OF CONFIDENCE LEVELS TO
INVESTIGATE, NO. OF REPLICATIONS, CONFIDENCE LEVELS, COMPONENT
RELIABILITIES, NO. OF TESTS PER COMPONENT, AND A CHI SQUARE DISTRIBUTION
TABLE.
LIST OF REFERENCES


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The purpose of this study is to evaluate the accuracy of a procedure used to compute an estimate of the lower 100(1-\(\gamma\))% confidence limit for reliability of a system of independent components connected in logical series. The procedure takes a Bayesian approach and uses test data on the individual components where the sample sizes may be unequal and no knowledge of the component failure distribution is needed. A computer simulation is used to generate test failure data and to compute estimates for the lower 100(1-\(\gamma\))% confidence limit on system reliability.
Reliability
Bayesian
Confidence Limit
Evaluation of the accuracy of a lower confidence limit estimate for series system reliability.
Evaluation of the accuracy of a lower co